Engineering Notes

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Model Reference Adaptive Identifier Design and Analysis

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I. Introduction

IN the past few years a considerable number of papers have appeared on linear systems identification. One promising approach is the model-reference adaptive system (MRAS) observer technique based on Lyapunov theory in order to insure convergence of the parameter estimates to the actual parameter values. Both continuous[1-3] and discrete deterministic observers,[4-6] as well as stochastic MRAS cases,[7,8] have been investigated. One of the hurdles to adaptive observer theory was the means of using only input-output data for a general nth order system to identify completely the pertinent system characteristics, but simultaneously to insure global asymptotic stability of the observer. A solution was found in the Meyer-Kalman-Yacubovitch (MKY) Lemma,[9] which was first used in adaptive control by Parks.[10] It was applied by Carroll[1] and Narendra[2] to the observer problem in order to eliminate all but a single-state variable from the computation of parameter estimator equations. Another approach uses Popov's Hyperstability Theory.[8]

In this Note, two new identification techniques for linear continuous systems identification are developed. Based on an adaptive observer proposed in Ref. 11, the new methods are obtained by developing a Lyapunov function with additional terms which will add design flexibility to the observer equations. The additional Lyapunov function terms lead to parameter estimator gains involving proportional and derivative factors in addition to the integral term in Ref. 11.

A linearized error analysis is performed in order to demonstrate the possible use of the new methods as regards design flexibility. Separate from this linearization analysis, asymptotic stability of the identifier is insured. The possible effects of noise, however, may limit the proposed schemes in practice due to differentiation.

II. Problem Statement

If a single-input, single-output linear time invariant system is controllable and observable, it may be parameterized in the form

$$\dot{x} = \left[\begin{array}{c|c} & H \\ \cline{2-2} a & \Lambda \end{array}\right] x + b + u \qquad y = Cx = x_1 \tag{1}$$

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with x an n-vector, $H = (1\ 1 \ldots 1)$, Λ an $(n-1)\times(n-1)$ diagonal matrix with constant negative real elements $-\lambda_i$, and a and b n-vectors of unknown parameters which are to be identified. A model for identifying a and b is

$$\dot{\hat{x}} = \left[\begin{array}{c|ccccc} \alpha_1 & 1 & 1 & 1 & \cdots & 1 \\ \cline{2-6} \alpha_2 & & & & & \\ \cdot & & & & & \\ \cdot & & & \Lambda & & \\ \cdot & & & & & \\ \alpha_n & & & & & \end{array}\right] \begin{bmatrix} x_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \cdot \\ \cdot \\ \hat{x}_n \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_n \end{bmatrix} u + \omega \tag{2}$$

where $\hat{x}$ is an nth order observer vector, and α_i and β_i are adjusted such that, as $t\to\infty$, $\phi = \alpha - a \to 0$, and $\psi = \beta - b \to 0$. The adaptation error is

$$e = \hat{x} - x \tag{3}$$

although only e_1 is physically measurable.

The term ω is an auxiliary input vector which approaches 0 as $t\to\infty$. Defining

$$\begin{aligned} \omega_1 &= -\lambda_1(\hat{x}_1 - x_1) \qquad \lambda_1 > 0 \\ \omega_i &= \mathcal{L}^{-1}\{(s+\lambda_i)^{-1}x_1(s)\}\dot{\phi}_i \\ &\quad + \mathcal{L}^{-1}\{(s+\lambda_i)^{-1}u(s)\}\dot{\psi}_i \qquad i = 2,3,\ldots n \end{aligned} \tag{4}$$

the error equation for $\dot{e}_1$ becomes[11]

$$\begin{aligned} \dot{e}_1 &= -\lambda_1 e_1 + x_1\phi_1 + [H\mathcal{L}^{-1}\{(sI-\Lambda)^{-1}x_1(s)\}\bar{\phi}] \\ &\quad + u\psi_1 + H\mathcal{L}^{-1}\{(sI-\Lambda)^{-1}u(s)\}\bar{\psi} + H\exp[\Lambda t]\bar{e}(t_0) \end{aligned} \tag{5}$$

where

$$\bar{e}(t_0) = (e_2, e_3, \ldots, e_n)\,|_{t_0}$$

and is the initial condition on the reduced order error vector,

$$\bar{\phi}^T = (\phi_2, \phi_3, \ldots, \phi_n),\ \bar{\psi}^T = (\psi_2, \psi_3, \ldots, \psi_n)$$

The design objective is to determine expressions for α and β requiring only the available information e_1 and x_1, such that $\alpha \to a$ and $\beta \to b$.

III. Derivation of Adaptive Identification Laws

Since an adaptive identification scheme is inherently nonlinear, proof of the convergence of the adaptive identification gains to the correct values is important since it is not obvious by inspection. To insure global parameter stability independent of the initial parameter guess, Lyapunov theory is used. The basic idea is to establish a Lyapunov function V, positive definite (p.d.) in e_1, ϕ, and ψ, evaluate the time derivative of the chosen V along the system trajectory, and design the identification gains in such a way that $\dot{V}$ is at least negative semidefinite, but as $t\to\infty$, $e\to 0$, $\phi\to 0$, and $\psi\to 0$.

To devise the two adaptation laws which guarantee global asymptotic stability in the $3n$ error space $\{e, \phi, \psi\}$, a

Lyapunov function candidate is selected to be

$$V=\frac{1}{2}e_1^2+\frac{1}{2}\sum_{i=1}^{n}\frac{1}{c_i}\left\{\phi_i+f_iL_i+\rho_i\frac{d}{dt}L_i\right\}^2+\frac{1}{2}\sum_{i=1}^{n}\rho_iL_i^2$$
$$+\frac{1}{2}\sum_{i=1}^{n}\frac{1}{d_i}\left\{\psi_i+g_iM_i+\sigma_i\frac{d}{dt}M_i\right\}^2+\frac{1}{2}\sum_{i=1}^{n}\sigma_iM_i^2 \qquad (6)$$

where c_i and d_i are constants>0, and f_i, g_i, ρ_i, and σ_i are constants ≥ 0. This Lyapunov function differs from the one in Ref. 11 in that there are additional p.d. terms associated with the former two cases. Selecting L_i, ϕ_i, and ψ_i as

$$L_1=e_1x_1 \qquad M_1=e_1u$$
$$L_i=e_1\mathcal{L}^{-1}\left\{\frac{x_1(s)}{s+\lambda_i}\right\} \qquad M_i=e_1\mathcal{L}^{-1}\left\{\frac{u(s)}{s+\lambda_i}\right\} \qquad (7)$$
$$\dot{\phi}_j=-c_jL_j-f_j\frac{d}{dt}L_j-\rho_j\frac{d^2}{dt^2}L_j$$
$$\dot{\psi}_j=-d_jM_j-g_j\frac{d}{dt}M_j-\sigma_j\frac{d^2}{dt^2} \qquad (8)$$

then $\dot{V}$ can be shown to reduce to

$$\dot{V}=-\lambda_1e_1^2+e_1H\exp[\Lambda t]\bar{e}(t_0)-\sum_{j=1}^{n}f_jL_j^2-\sum_{j=1}^{n}g_jM_j^2 \qquad (9)$$

Defining $z(t)=H\exp[\Lambda t]\bar{e}(t_0)$, then

$$\dot{V}=-\lambda_1e_1^2-\sum_{j=1}^{n}f_jL_j^2-\sum_{j=1}^{n}g_jM_j^2+e_1z(t)\leq e_1z(t)\leq\sqrt{V}|z(t)| \qquad (10)$$

Now $z(t)\to 0$ exponentially fast as $t\to\infty$, and $\dot{V}\to 0$, implying $e_1\to 0$ as $t\to\infty$. It has been shown[1] that $e\to 0$ is not a sufficient condition for insuring asymptotic stability of $\{e, \phi, \psi\}$. However, if u contains at least n distinct frequencies (i.e., is sufficiently "frequency rich"),[1,11] then $e_1=0\to\phi=\psi\equiv\mathbf{0}$.

The vector ω can now be explicitly determined. Defining

$$v_i=\mathcal{L}^{-1}\left\{\frac{x_1(s)}{s+\lambda_i}\right\} \qquad s_i=\mathcal{L}^{-1}\left\{\frac{u(s)}{s+\lambda_i}\right\} \quad i=2,3,...n \qquad (11)$$

and using Eq. (8), ω_1 in Eq. (4) becomes

$$\omega_i=-c_iv_i^2-d_is_i^2-f_iv_i\frac{d}{dt}(e_1v_i)-g_is_i\frac{d}{dt}(s_ie_1)$$
$$-\rho_iv_i\frac{d^2}{dt^2}(e_1v_i)-\sigma_is_i\frac{d^2}{dt^2}(e_1s_i) \qquad (12)$$

The terms v_i and s_i represent "pseudostates and inputs" replacing the vector x and derivatives of u, all except x_1 and u of which are physically unavailable. Therefore, this identifier actually requires n states and n inputs of information, however ingeniously it is indirectly obtained. From Eq. (8), the adaptive "gains," or parameter identification equations, become

$$\alpha_1=-c_1\int_{t_0}^{t}e_1x_1dt-f_ie_1x_1-\rho_i\frac{d}{dt}(e_1x_1) \qquad (13)$$

$$\alpha_i=-c_i\int_{t_0}^{t}e_1v_idt-f_ie_1v_i-\rho_i\frac{d}{dt}(e_1v_i) \qquad (14)$$

$$\beta_1=-d_1\int_{t_0}^{t}e_1u\,dt-g_1e_1u-\sigma_1\frac{d}{dt}(e_1u) \qquad (15)$$

$$\beta_i=-d_i\int_{t_0}^{t}e_1s_idt-g_ie_1s_i-\sigma_i\frac{d}{dt}(e_1s_i) \qquad (16)$$

$$c_i,d_i,\lambda_i>0, \quad \text{and} \quad f_ig_i,\rho_i,\sigma_i\geq 0,\ i=2,3,...,n$$

Two different cases result from using Eq. (6), one if f_i, g_i, ρ_i, $\sigma_i>0$, and another if only $\rho_i=\sigma_i=0$. Since the two identification approaches require more hardware than the method of Ref. 11, the question arises as to what the tradeoffs and benefits are in using such approaches. One of the possible advantages will be shown to include improved convergence rate, while a possible disadvantage is the effect of noise on the derivative terms in Eqs. (12-16).

IV. Comparison of the Identification Laws

To compare the two approaches previously developed to the method in Ref. 11, an appropriate error convergence rate analysis will be performed. If it can be shown that the proposed laws exhibit improved transient response, then they can be considered superior under certain conditions. It has previously been shown that V and $\dot{V}$ can be used to estimate the system convergence rate.[12,13] Also a linearized error characteristic equation (LECE) approach has been used.[14-16] Some recent papers[17,18] provide some exact results for limited cases. For purposes of a simple comparison of methods, the LECE approach is employed here. For illustration purposes, only a second-order case is employed.

The LECE for the last method presented will be developed, since the other two identification laws are special cases of this method. Consider a second-order plant-model system like Eqs. (1) and (2). Generalizing $\dot{e}_1$, $\dot{\phi}_i$, $\dot{\psi}_i$, linearizing about an assumed operating point denoted by o,

$$x_1=x_1^o,\ v_2=v_2^o,\ u=u^o,\ s_2=s_2^o,\ \phi_i=\phi_i^o,\ \psi_i=\psi_i^o=0 \qquad (17)$$

and truncating after linear terms yields a set of linearized state equations. Solving the coupled reduced linear equations using Laplace transforms and substituting into the perturbation equation for $\dot{e}_1$ results in the LECE for $\Delta E_1(s)$:

$$1+(K_1+K_2s+K_3s^2)/[s(s+\lambda_1)]=0 \qquad (18)$$

where

$$K_1=c_1x_1^{o^2}+c_2v_2^{o^2}+d_1u^{o^2}+d_2s_2^{o^2}$$
$$K_2=f_1x_1^{o^2}+f_2v_2^{o^2}+g_1u^{o^2}+g_2s_2^{o^2}$$
$$K_3=\rho_1x_1^{o^2}+\rho_2v_2^{o^2}+\sigma_1u^{o^2}+\sigma_2s_2^{o^2} \qquad (19)$$

In the derivation of Eq. (18), it is assumed that a constant input u has been applied for a "long" time (so the time derivatives will be zero). It should be noted that this analysis cannot be applied strictly to the identification problem because it was assumed u is constant, but a necessary condition for identification was that u possess at least n distinct frequencies. Without making the assumption of constant inputs, meaningful results are not possible for the LECE approach. As long as the time derivative terms ($f_1\dot{x}_1$, $\rho_1\dot{e}_1$, etc.) are much less than the "steady-state" terms (c_1x_1, c_2v_2, d, u, g_2s_2, etc.) then the analysis will be meaningful. One way to accomplish this is to bias u such that the bias tends to dominate ac variations, where the ac signals are of low enough frequencies so that the time derivatives are "small."

Note that Eq. (18) is in the general form $1+kP(s)=0$ for root locus analysis. Comparing Eq. (8) with Eq. (18), it can be seen that the gains K_i are like a proportional-integral-derivative (P-I-D) controller, analogous to Ref. 19. If $\rho_i=\sigma_i=0$, the identifier is a P-I type system analogous to Ref. 20, since $K_3=0$. If, in addition to $\rho_i=\sigma_i=0$, $f_i=g_i=0$, then $K_2=K_3=0$, and the identifier is an I-type. Comparable to the *control* case of Ref. 21. These results further demonstrate the basic realizability of the triplet grouping of the MRAS adaptation rules in Refs. 19-21. The existence of this triplet sequence is further shown by the work of Shahein,[22] Ten Cate,[23] and Colburn and Boland.[24]

The additional terms in Eq. (8) can allow for increased flexibility in error convergence rate control. By proper selection of the f, g, ρ, and σ terms, an improved convergence rate could result. This could be of great importance if one wishes to use the identifier results for further control operations or where stability information is crucial, as in power systems. It has been shown[25,26] that for complete analysis, interpretation of a criterion surface in $2n$ dimension parameter space is needed. However, such work is beyond the scope of this Note. Also, although Eqs. (13-16) require differentiation, which is undesirable from a noise viewpoint, variations in filtering and problem structure modification could be employed for purposes of implementation.

V. Conclusions

Two new adaptive identification laws were developed and asymptotic stability proved using Lyapunov theory. Using an approximate analysis employing a linearized error characteristic equation approach, it was shown how the two methods are analogous to a previously published method as regards classical P-I-D type control laws. Due to differentiation, if noise is present there may be practical problems with regard to tracking accuracy and stability. An advantage of the two methods presented is the possible improvement in the transient tracking response, as shown by a linearized error response analysis.

Some interesting areas for future work include: 1) determining an exact transient stability analysis approach, as in Refs. 17 and 25, and 2) determining the effect of relaxing the "frequency richness" input requirement on the size of the parameter tracking errors $\|\phi\|$, $\|\psi\|$ (since asymptotic stability of the parameter estimates is no longer assured).[27]

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Stability Augmentation by Eigenvalues Control and Model Matching

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Introduction

THE high-performance requirements of modern aircraft have led to the development of digital fly-by-wire control systems based on model-following techniques.[1-4] In such techniques, a model embodying all desired performance

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